

**Whole School Written Calculation Policy**  
**Pencil and paper procedures**  
**Key Stages 1 - 2**  
**St Peter and St Paul Primary School,**  
**Carbrooke**

Review date:

# **St Peter and St Paul,Carbrooke policy for Pencil and paper procedures in Numeracy**

## ***Background to the policy***

This policy contains the key pencil and paper procedures that will be taught within our school. It has been written to ensure consistency and progression throughout the school and reflects a whole school agreement.

Although the focus of the policy is on pencil and paper procedures it is important to recognise that the ability to calculate mentally lies at the heart of the Primary National Strategy for mathematics. The mental methods in the *Primary Framework for teaching mathematics* will be taught systematically from Reception onwards and pupils will be given regular opportunities to develop the necessary skills. However mental calculation is not at the exclusion of written recording and should be seen as complementary to and not as separate from it. In every written method there is an element of mental processing. Sharing written methods with the teacher encourages children to think about the mental strategies that underpin them and to develop new ideas. Therefore written recording both helps children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies.

During their time at this school children will be encouraged to see mathematics as both a written and spoken language. Teachers will support and guide children through the following important stages:

- developing the use of pictures and a mixture of words and symbols to represent numerical activities;
- using standard symbols and conventions;
- use of jottings to aid a mental strategy;
- use of pencil and paper procedures;
- use of a calculator.

This policy concentrates on the introduction of standard symbols, the use of the empty number line as a jotting to aid mental calculation and on the introduction of pencil and paper procedures. It is important that children do not abandon jottings and mental methods once pencil and paper procedures are introduced. Therefore children will always be encouraged to look at a calculation/problem and then decide which is the best method to choose – pictures, mental calculation with or without jottings, structured recording or a calculator. Our long-term aim is for children to be able to select an efficient method of their choice (whether this be mental, written or in upper Key Stage 2 using a calculator) that is appropriate for a given task. They will do this by always asking themselves:

**‘Can I do this in my head?’**

**‘Can I do this in my head using drawings or jottings?’**

**‘Do I need to use a pencil and paper procedure?’**

***‘Do I need a calculator?’***

▪

**Addition**

**Year 1**

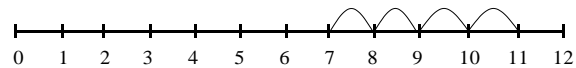
**+ = signs and missing numbers**

$$\begin{array}{ll}
 3 + 4 = \square & \square = 3 + 4 \\
 3 + \square = 7 & 7 = \square + 4 \\
 \square + 4 = 7 & 7 = 3 + \square \\
 \square + \nabla = 7 & 7 = \square + \nabla
 \end{array}$$

Promoting covering up of operations and numbers.

**Number tracks and number lines (numbered)**

$$7 + 4$$



Recording by - drawing jumps on prepared lines

- constructing own lines

(Teacher model number lines with missing numbers)

(Teachers model jottings appropriate for larger numbers)

**Year 2**

**+ = signs and missing numbers**

Continue using a range of equations as in Year 1 but with appropriate, larger numbers.

Extend to

$$14 + 5 = 10 + \square$$

and adding three numbers

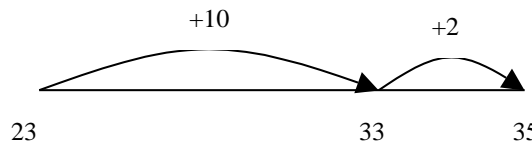
$$32 + \square + \square = 100 \quad 35 = 1 + \square + 5$$

**Partition into tens and ones and recombine**

$$\begin{aligned}
 12 + 23 &= 10 + 2 + 20 + 3 \\
 &= 30 + 5 \\
 &= 35
 \end{aligned}$$

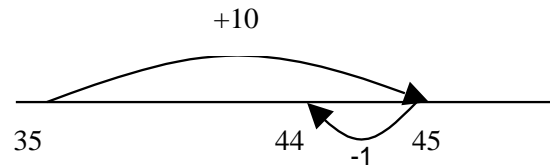
refine to partitioning the second number only:

$$\begin{aligned}
 23 + 12 &= 23 + 10 + 2 \\
 &= 33 + 2 \\
 &= 35
 \end{aligned}$$



Add 9 or 11 by adding 10 and adjusting by 1

$$35 + 9 = 44$$



**Year 3**

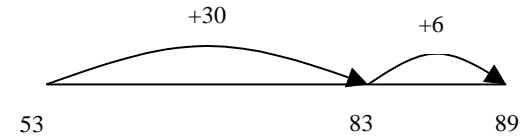
**+ = signs and missing numbers**

Continue using a range of equations as in Year 1 and 2 but with appropriate, larger numbers.

**Partition into tens and ones and recombine**

Partition both numbers and recombine. Refine to partitioning the second number only e.g.

$$\begin{aligned}
 36 + 53 &= 53 + 30 + 6 \\
 &= 83 + 6 \\
 &= 89
 \end{aligned}$$



**Add a near multiple of 10 to a two-digit number**

Continue as in Year 2 but with appropriate numbers e.g.  $35 + 19$  is the same as  $35 + 20 - 1$ .

**pencil and paper procedures**

$$83 + 42 = 125$$

<b>G&amp;T</b>		
$80 + 3$	$83$	$83$
$+40 + 2$	$+ 42$	$+ 42$
$120 + 5 = 125$	$120$	$5$
	$\underline{\quad 5}$	$\underline{120}$
	<b>125</b>	<b>125</b>

## Addition

### Year 4

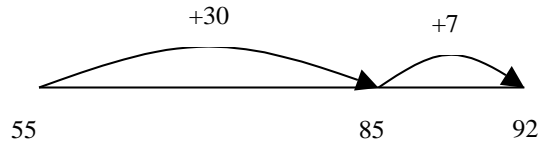
**+ = signs and missing numbers**

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

**Partition into tens and ones and recombine**

Either partition both numbers and recombine or partition the second number only e.g.

$$\begin{aligned} 55 + 37 &= 55 + 30 + 7 \\ &= 85 + 7 \\ &= 92 \end{aligned}$$



**Add the nearest multiple of 10, then adjust**

Continue as in Year 2 and 3 but with appropriate numbers e.g.  $63 + 29$  is the same as  $63 + 30 - 1$

**Pencil and paper procedures**

$$358 + 73 = 431$$

either                      or

$\begin{array}{r} 300+50+8 \\ + \quad 70+3 \\ \hline 300+120+11 = 431 \end{array}$	$\begin{array}{r} 358 \\ \quad 73 \\ \hline 300 \\ 120 \\ 11 \\ \hline 431 \end{array}$
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Extend to decimals in the context of money (vertically)

$$£ 2.50 + £ 1.75 = £ 4.25$$

$$\begin{array}{r} £ 2.50 \\ + £ 1.75 \\ \hline £ 3.00 \\ £ 1.20 \\ \hline £ 0.05 \\ \hline £ 4.25 \end{array}$$

1

### Year 5

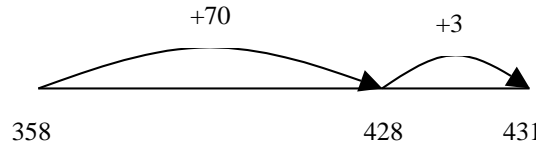
**+ = signs and missing numbers**

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

**Partition into hundreds, tens and ones and recombine**

Either partition both numbers and recombine or partition the second number only e.g.

$$\begin{aligned} 358 + 73 &= 358 + 70 + 3 \\ &= 428 + 3 \\ &= 431 \end{aligned}$$



**Add or subtract the nearest multiple of 10 or 100, then adjust**

Continue as in Year 2, 3 and 4 but with appropriate numbers e.g.  $458 + 79 =$  is the same as  $458 + 80 - 1$

**Pencil and paper procedures**

$$\begin{array}{r} 358 \\ \quad 73 \\ \hline 300 \\ 120 \\ 300 \\ 431 \end{array}$$

Extend to decimals (same number of decimal places) and adding several numbers (with different numbers of digits).

*Model negative numbers using a number line.*

### Year 6

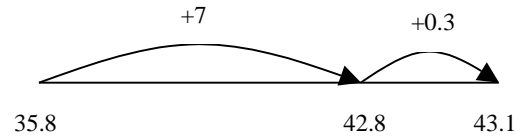
**+ = signs and missing numbers**

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

**Partition into hundreds, tens, ones and decimal fractions and recombine**

Either partition both numbers and recombine or partition the second number only e.g.

$$\begin{aligned} 35.8 + 7.3 &= 35.8 + 7 + 0.3 \\ &= 42.8 + 0.3 \\ &= 43.1 \end{aligned}$$



**Add the nearest multiple of 10, 100 or 1000, then adjust**

Continue as in Year 2, 3, 4 and 5 but with appropriate numbers including extending to adding 0.9, 1.9, 2.9 etc

**Pencil and paper procedures**

Extend to numbers with any number of digits and decimals with 1 and 2 decimal places.

$$124.9 + 117.25 = 242.15$$

$$\begin{array}{r} 124.9 \\ + 117.25 \\ \hline 200.00 \\ 30.00 \\ 11.00 \\ 1.10 \\ 0.05 \\ \hline 242.15 \end{array}$$

**Leading to formal method, showing numbers carried underneath only for G&T children.**

$$\begin{array}{r} 358 \\ + 73 \\ \hline 431 \\ \hline \end{array}$$

Extend to numbers with at least four digits

$$3587 + 675 = 4262$$

$$\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \\ \hline \end{array}$$

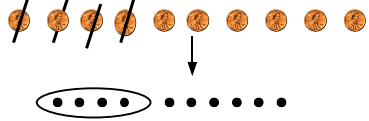
Revert to expanded methods if the children experience any difficulty.

## Subtraction find the difference

### Year 1

**Pictures / marks**

Sam spent 4p. What was his change from 10p?  
Find the difference in value of the two sets



**- = signs and missing numbers**

$$7 - 3 = \square \quad \square = 7 - 3$$

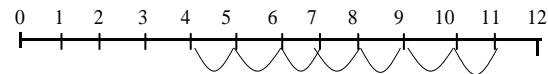
$$7 - \square = 4 \quad 4 = \square - 3$$

$$\square - 3 = 4 \quad 4 = 7 - \square$$

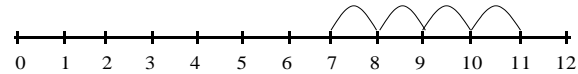
$$\square - \nabla = 4 \quad 4 = \square - \nabla$$

**Number lines (numbered)**

$11 - 7$   
(Counting on to find the difference in value)



The difference between 7 and 11  
(Counting up)



Recording by - drawing jumps on prepared lines  
- constructing own lines

(Teachers model jottings appropriate for larger numbers)

### Year 2

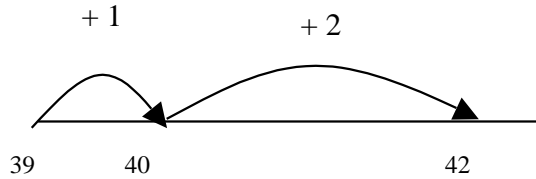
**- = signs and missing numbers**

Continue using a range of equations as in Year 1 but with appropriate numbers.

Extend to  $14 + 5 = 20 - \square$

Find a small difference by counting up

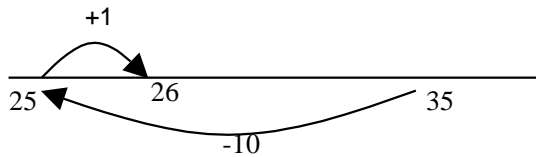
$$42 - 39 = 3$$



*Subtract 9 or 11. Begin to add/subtract 19 or 21*

$$35 - 9 = 26$$

Children taught to look at the numbers and to decide if they would be better off counting up or counting back.



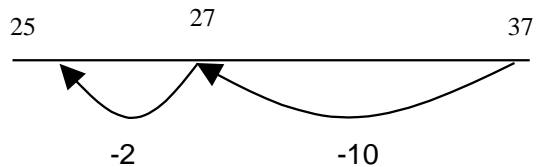
**Use known number facts and place value to subtract**

(partition second number only)

$$37 - 12 = 37 - 10 - 2$$

$$= 27 - 2$$

$$= 25$$



### Year 3

**- = signs and missing numbers**

Continue using a range of equations as in Year 2 but with appropriate numbers.

**Find a small difference by counting up**

Continue as in Year 2 but with appropriate numbers e.g.  $102 - 97 = 5$  use find the difference

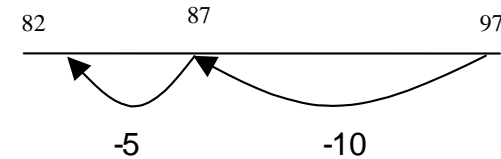
Or  $102 - 19 = 83$  do  $102 - 20 + 1 = 83$

**Subtract mentally a 'near multiple of 10' to or from a two-digit number**

Continue as in Year 2 but with appropriate numbers e.g.  $78 - 49$  is the same as  $78 - 50 + 1$

**Use known number facts and place value to subtract**

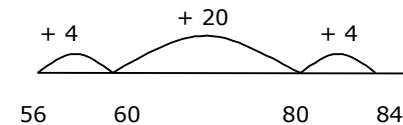
Continue as in Year 2 but with appropriate numbers e.g.  $97 - 15 = 72$



**Pencil and paper procedures**

Complementary addition

$$84 - 56 = 28$$



## Subtraction

### Year 4

**- = signs and missing numbers**

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

*Find a small difference by counting up*

e.g.  $5003 - 4996 = 7$

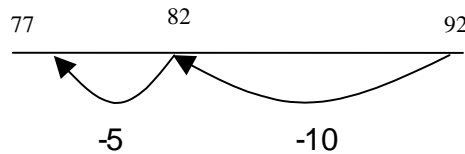
This can be modelled on an empty number line (see complementary addition below).

**Subtract the nearest multiple of 10, then adjust.**

Continue as in Year 2 and 3 but with appropriate numbers.

**Use known number facts and place value to subtract**

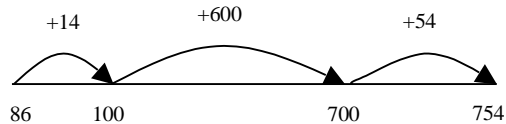
$92 - 15 = 67$



**Pencil and paper procedures**

Complementary addition

$754 - 86 = 668$



### Year 5

**- = signs and missing numbers**

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

*Find a difference by counting up*

e.g.  $8006 - 2993 = 5013$

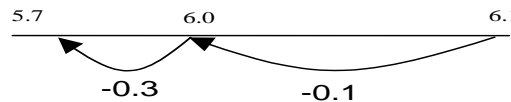
This can be modelled on an empty number line (see complementary addition below).

**Subtract the nearest multiple of 10 or 100, then adjust.**

Continue as in Year 2, 3 and 4 but with appropriate numbers.

**Use known number facts and place value to subtract**

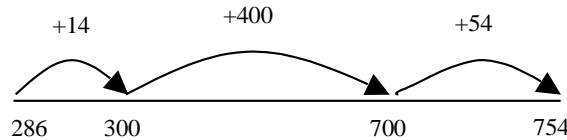
$6.1 - 0.4 = 5.7$



Pencil and paper procedures

Complementary addition

$754 - 286 = 468$



**OR**       $754 - 286 = 468$

754		
<u>  - 286</u>		
14 (300)	can be refined to	14 (300)
400 (700)		454 (754)
<u>  54 (754)</u>		468
468		

### Year 6

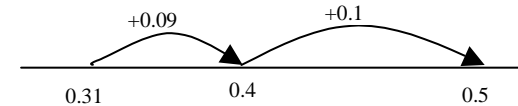
**- = signs and missing numbers**

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

*Find a difference by counting up*

e.g.  $0.5 - 0.31 = 0.19$

This can be modelled on an empty number line (see complementary addition below).



**Subtract the nearest multiple of 10, 100 or 1000, then adjust**

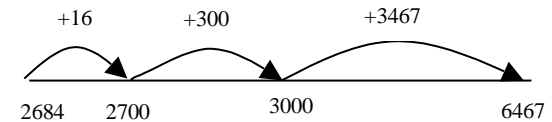
Continue as in Year 2, 3, 4 and 5 but with appropriate numbers.

Use known number facts and place value to subtract  
Continue as year 5

**Pencil and paper procedures**

Complementary addition

$6467 - 2684 = 3783$



**OR**       $6467 - 2684 = 3783$

6467		
<u>  - 2684</u>		
16 (2700)	can be refined to	316 (3000)
300 (3000)		3467 (6467)
<u>  3467 (6467)</u>		3783
3783		

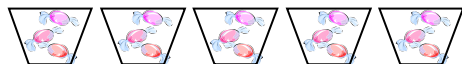
(Decomposition for G&T children only when secure.)

## Multiplication and Division Teach integrally

### Year 1

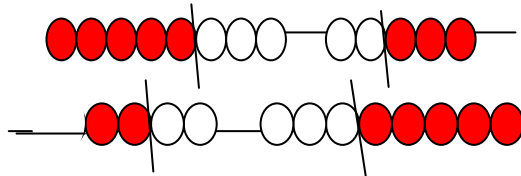
#### Pictures and symbols

There are 3 sweets in one bag.  
How many sweets are there in 5 bags?



*(Recording on a number line modelled by the teacher when solving problems)*

Use of bead strings to model groups of.



### Year 2

#### x = signs and missing numbers

$$7 \times 2 = \square \quad \square = 2 \times 7$$

$$7 \times \square = 14 \quad 14 = \square \times 7$$

$$14 \div 7 = \square \quad 14 \div 2 = \square$$

$$14 \div \square = 2 \quad 14 \div \square = 7$$

$$\square \times 2 = 14 \quad 14 = 2 \times \square$$

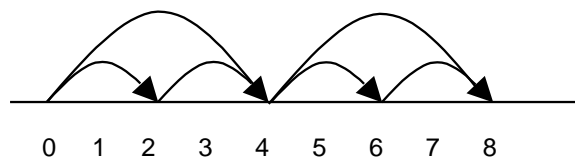
$$\square \times \nabla = 14 \quad 14 = \square \times \nabla$$

#### Arrays and repeated addition

$4 \times 2$  or  $4 + 4$   
 $2 \times 4$

Or  $8 \div 4$  or  $8 \div 2$   
or repeated addition

$$2 + 2 + 2 + 2$$



#### Doubling multiples of 5 up to 50

$$15 \times 2 = 30$$

#### Partition

~~$$15 \times 2$$~~

$$20 + 10 = 30$$

**OR**

x	10	5
2	20	10

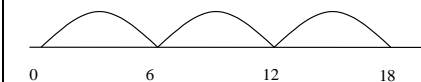
### Year 3

#### x = signs and missing numbers

Continue using a range of equations as in Year 2 but with appropriate numbers.

Number lines

$$6 \times 3$$



*Arrays and repeated addition*

Continue to understand multiplication as repeated addition and continue to use arrays (as in Year 2).

#### **Doubling multiples of 5 up to 50**

$$35 \times 2 = 70$$

Partition

x	30	5
2	60	10

Use known facts and place value to carry out simple multiplications

Use the same method as above (partitioning), e.g.  $32 \times 3$

$$= 96$$

x	30	2
3	90	6



## Multiplication

### Year 4

**x = signs and missing numbers**

Continue using a range of equations as in Year 2 but with appropriate numbers

Use the grid method of multiplication (as below)

**Pencil and paper procedures**

Grid method

23 x 7 is approximately 20 x 10 = 200

x	20	3
7	140	21

x	70	2
30	2100	60
8	560	16

### Year 5

**x = signs and missing numbers**

Continue using a range of equations as in Year 2 but with appropriate numbers

Use the grid method of multiplication (as below)

**Pencil and paper procedures**

Grid method

72 x 38 is approximately 70 x 40 = 2800

x	70	2
30	2100	60
8	560	16

Extend to simple decimals with one decimal place.

### Year 6

**x = signs and missing numbers**

Continue using a range of equations as in Year 2 but with appropriate numbers

Use the grid method of multiplication (as below)

**Pencil and paper procedures**

**Grid method**

372 x 24 is approximately 400 x 20 = 8000

x	300	70	2
20	6000	1400	40
4	1200	280	8

Extend to decimals with up to two decimal places.

## Division and Multiplication Teach integrally

Year 1

**Pictures / marks**

12 children get into teams of 4 to play a game.  
How many teams are there?



Year 2

**÷ = signs and missing numbers**

$6 \div 2 = \square$	$\square = 6 \div 2$
$6 \div \square = 3$	$3 = 6 \div \square$
$2 \times 3 = \square$	$3 \times 2 = \square$
$\square \times 3 = 6$	$2 \times \square = 6$
$\square \div 2 = 3$	$3 = \square \div 2$
$\square \div \nabla = 3$	$3 = \square \div \nabla$

**Understand division as sharing and grouping**

Sharing – 6 sweets are shared between 2 people. How many do they have each?



Can be written as

$6 \div 2 = 3$   
 $2 \times \square = 6$

$6 \div 2$  can be modelled as:

Grouping – There are 6 sweets. How many people can have 2 each? (How many 2's make 6?)



Year 3

**÷ = signs and missing numbers**

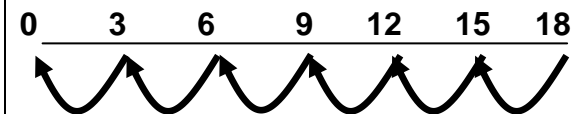
Continue using a range of equations as in Year 2 but with appropriate numbers.

**Understand division as sharing and grouping**

$18 \div 3$  can be modelled as:

Sharing – 18 shared between 3 (see Year 2 diagram)

OR



Or

Grouping - How many 3's make 18? or  $3 \times \square = 18$

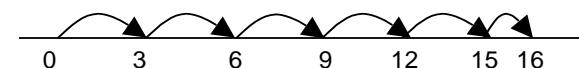


***Remainders***

$16 \div 3 = 5 \text{ r}1$

Sharing - 16 shared between 3, how many left over?

Grouping – How many 3's make 16, how many left over?  
e.g.



## Division

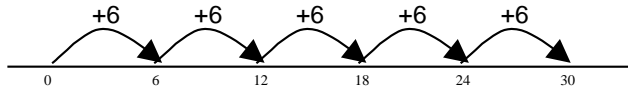
### Year 4

**÷ = signs and missing numbers**

Continue using a range of equations as in Year 2 but with appropriate numbers.

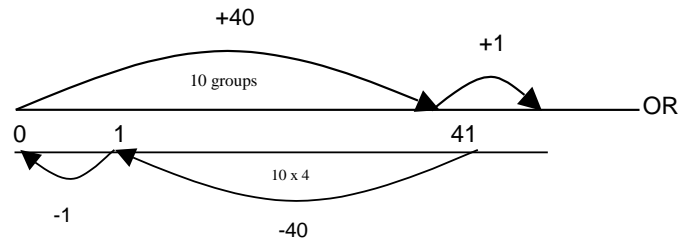
**Sharing and grouping**

30 ÷ 6 can be modelled as:  
grouping – groups of 6 taken away and the number of groups counted e.g.



sharing – sharing among 6, the number given to each person

Remainders  
41 ÷ 4 = 10 r1



OR 41 = (10 x 4) + 1

**Pencil and paper procedures**

72 ÷ 5 lies between 50 ÷ 5 = 10 and 100 ÷ 5 = 20

$$\begin{array}{r} 72 \\ - 50 \quad (10 \text{ groups}) \text{ or } (10 \times 5) \\ \hline 22 \\ - 20 \quad (4 \text{ groups}) \text{ or } (4 \times 5) \\ \hline 2 \end{array}$$

Answer : 14 remainder 2

### Year 5

**÷ = signs and missing numbers**

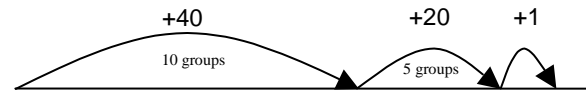
Continue using a range of equations as in Year 2 but with appropriate numbers.

**Sharing and grouping**

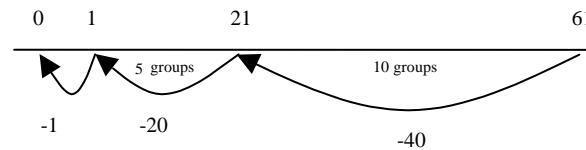
Continue to understand division as both sharing and grouping (repeated subtraction).

**Remainders**

Quotients expressed as fractions or decimal fractions  
61 ÷ 4 = 15 ¼ or 15.25



OR



**Pencil and paper procedures**

256 ÷ 7 lies between 210 ÷ 7 = 30 and 280 ÷ 7 = 40

$$\begin{array}{r} 256 \\ - 70 \quad (10 \text{ groups}) \text{ or } (10 \times 7) \\ \hline 186 \\ - 140 \quad (20 \text{ groups}) \text{ or } (20 \times 7) \\ \hline 46 \\ - 42 \quad (6 \text{ groups}) \text{ or } (6 \times 7) \\ \hline 4 \quad (36 \text{ groups}) \text{ or } (36) \end{array}$$

Answer: 36 remainder 4

### Year 6

**÷ = signs and missing numbers**

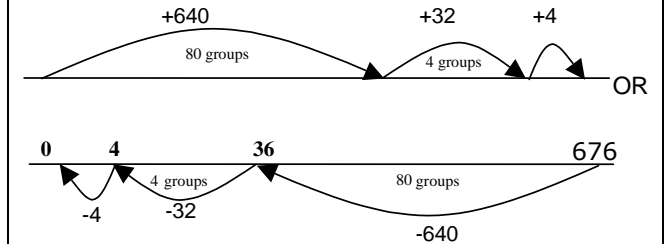
Continue using a range of equations as in Year 2 but with appropriate numbers.

**Sharing and grouping**

Continue to understand division as both sharing and grouping (repeated subtraction).

**Remainders**

Quotients expressed as fractions or decimal fractions  
676 ÷ 8 = 84.5



**Pencil and paper procedures**

977 ÷ 36 is approximately 1000 ÷ 40 = 25

$$\begin{array}{r} 977 \\ - 360 \quad (10 \text{ groups}) \\ \hline 617 \\ - 257 \quad (5 \text{ groups}) \\ \hline 360 \quad (10 \text{ groups}) \text{ refine to } \\ - 257 \quad (5 \text{ groups}) \\ \hline 180 \quad (5 \text{ groups}) \\ - 77 \quad (2 \text{ groups}) \\ \hline 72 \quad (2 \text{ groups}) \\ - 72 \\ \hline 5 \end{array}$$

Answer: 27 5/36

(Formal method to continue to be taught to present Y6 04 – 05)

## Addition

### Vocabulary

It is essential that all staff use the same vocabulary when teaching +. The key phrase to use is *find the total value of the two amounts* when explaining to children what the + symbol means. We can then go on to explain that find the total is not always the words we hear when we have to solve + problems...these are add plus sum etc and that we can find the total by counting forwards.

All children must + using number lines or number tracks by counting on from the biggest value number. However, children need to be aware that addition can be done in any order but that by starting with the largest and adding on to the smallest value digit is usually the more efficient method.

### **Reception**

Picture track use everyday language forward add on, count on, etc

Ensure the word total value is brought in when doing one to one correspondence.

Adding together groups of objects such as I have two red cars and three blue cars, how many do I have altogether?

Bring in the word value when you write the digits e.g. say to the children, *'let's write that down, the value of this group is 5 and the value of this group is 3. What's the total value of the two groups?'*

### **Year 1**

Finger addition

Put the largest number in your head and add on the smallest number.

Also use cubes to add groups of objects.

Leading onto the introduction of adding along a number track.

### **Year 2**

Use number lines to add along leading to the end of the year children being able to add on a blank number line which has been provided for them. Begin to draw empty number lines in summer term with some children being able to draw their own

### **Year 3**

Children using empty number lines and drawing their own.

Follow the teaching sequences in the renewed framework but not teaching composition in upper KS2.

#### **Essential knowledge**

##### **Number patterns**

It is important that children know the number patterns on dice dominoes and cards for the numbers to 10 this will support them in their mental – and + problems.

Children must know number pairs for 2,3,4,5,6,7,8,9,10.20.100. These are essential for children's knowledge of – and +. These must be taught in Foundation and KS1 and KS2 if necessary.

Count in ones, tens and hundreds

Doubles and near doubles

Number pairs and near number pairs

#### **Essential vocabulary**

Total

Sum

Altogether

Count on

Add

Plus

Etc

## Subtraction

(do not teach decomposition anywhere!!)

### Vocabulary

It is essential that all staff use the same vocabulary when teaching -. The key phrase to use is *find the difference in value of the two amounts* when explaining to children what the – symbol means. We can then go onto explain that find the difference is not always the words we hear when we have to solve – problems...these are minus, take away, less etc and that we can find the difference by counting forwards or backwards.

All children must – using number lines or number tracks by counting on from the smallest value number to the largest value number.

### **Reception**

Picture track use everyday language forward backward onto etc

Ensure the word value is brought in when doing one to one correspondence.

Groups of objects and use language how many more how many less

e.g. there are 5 children in this group, there are 3 children in this group, how many more children are there in this group etc.

Bring in the word value when you write the digits e.g. let's write that down the value of this group is 5 and the value of this group is 3 what's the difference in value between the two groups

Then bring in cubes in towers what's the difference in value of the two towers? Build up to this by doing the following:

Lay out a group of 6 individual cubes and a group of 3 individual cubes, say to the chn we are going to find the difference in value of these two groups but it's quite tricky with them in groups so I'm going to make a tower. Lay towers side by side. Do this practically then record number sentence alongside the towers  $6-3=3$  say what is the difference in value between a tower of 6 and a tower of 3. This then leads onto the use of number tracks/lines. (Number tracks are used until the child is ready to move to a number line, around about the end of yr1 start of yr2, fine motor skills at using lines instead of boxes and must have secure number value knowledge)

### Numbers on a track:

When children are secure with finding the difference using towers go onto the number track. Always begin with the smallest number and count up.

Round to nearest ten then count on in tens then units. This is essential when children begin to deal with large calculations.

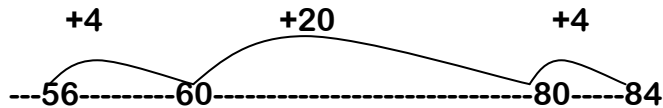
This means children must be secure in number pairs to 10, near number pairs, doubling and near doubles, to enable them to add up the units.

e.g.  $37 - 16 = 21$



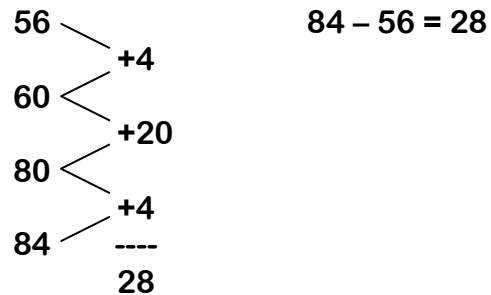
In year 3 children continue to use no line to – but teacher scribes the falling zig zag next to the number line, until children are totally secure with finding the difference on a number line they won't be able to use the zig zag tower, as their understanding develops and they are constantly exposed to the zig zag tower they will be able to incorporate it alongside their number line with the aim that by yr 6 they should be able to use a refined version of the tower (complimentary addition) see Numeracy strategy folder for examples.

### Using a number line



Vertical zig zag tower

*(counting up from 56 to 84)*



### Complimentary addition

This must only be taught when the children are secure with subtraction on a number line

### Complementary addition using vertically aligned digits

*(counting up from 86 to 754)*

$$\begin{array}{r} 754 \\ - 86 \\ \hline 4 \text{ to count up to } 90 \\ 10 \text{ to count up to } 100 \\ 600 \text{ to count up to } 700 \\ 50 \text{ to count up to } 750 \\ 4 \text{ to count up to } 754 \\ \hline 668 \end{array}$$

### Subtracting 19 using the compensation method

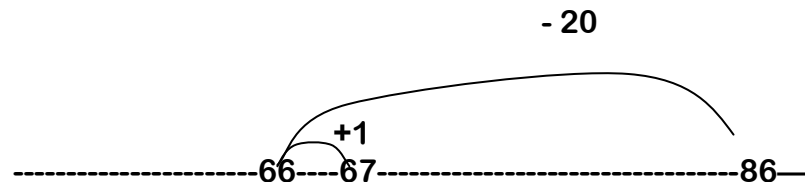
*(take too much, add back)*

#### Example 1

The children subtract twenty, then adjust by adding one.

The children need to be secure with their number pairs to ten and twenty to understand this technique.

$$86 - 19 = 86 - 20 + 1$$

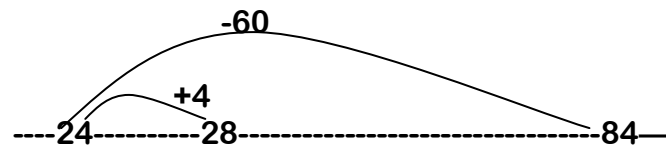


This technique can be used with higher value digits as well.



## Example 2

$$84 - 56 = 84 - 60 + 4$$



Subtract 60 then add on 4.

Children need to be confident with rounding digits to their nearest ten for this type of subtraction.

### Compensation using vertically aligned digits

*(Take too much, add back)*

$$\begin{array}{r} 754 \\ - 86 \\ \hline 654 \quad (754 - 100) \\ + 14 \quad (\text{since } 100 - 86 = 14) \\ \hline \underline{668} \end{array}$$

Instead of subtracting 86 subtract 100

Next find the number pair to go with 86 and add it to the previous stage

### Essential knowledge

#### Number patterns

It is important that children know the number patterns on dice dominoes and cards for the numbers to 10 this will support them in their mental – and + problems.

Children must know number pairs for 2,3,4,5,6,7,8,9,10,20,100. These are essential for chn's knowledge of – and +. These must be taught in Foundation and KS1 and ks2 if necessary.

Count in ones, tens and hundreds

Doubles and near doubles

Number pairs and near number pairs

### Essential vocabulary

Find the difference in value

This can also be described in word problems as:

Take away

Subtract

Less than

Count back

Minus

When we read a number sentence such as  $9-5=4$  we would say find the difference in value between 9 and 5 but in word problems we would encounter vocabulary such as there are 9 cars in a car park, 5 leave how many are left? We are actually being asked to find the value between the original 9 cars and the remaining 5.

### Multiplication Division

#### Essential vocabulary

x

Sets of

Groups of

Rows of

Lots of

Times

Multiples

Factors

Product

Within word problems children need to recognise that phrases such as two plates with four apples how many apples and five packs of 6 eggs means to multiply.

÷

Share

Equal

Sets of

Groups of

Rows of    Lots of    Quotient

**Multiplication and division must be taught together as should doubling and halving which is the beginning stages of this process.**

### **Tables**

**Children must count on a daily basis both up and down and from different starting points.**

**Blank number sticks to be used to count along when introducing a new table**

**Children help to identify numbers on the counting stick using doubling and halving and combining**

**Counting in tens and fives on 100 square to identify pattern**

### **Year 1**

**Introduce counting in 2's,10's then 5's.**

**Counting in 2's relate to odd and even**

**Counting in 10's flash ten fingers as they say the words**

**Counting in 5's flash five fingers as they say the words**

### **Doubling and halving**

**In Year 1 the children learn the connection between doubling and halving and multiplying or dividing by 2.**

**They begin to double/halve by physically being given 3 bricks then taking three more, or by starting with 8 books and sharing them out between 2 people.**

**Initially these activities are all carried out as practically as possible, which allows the concept to be grounded in reality. As they gain in understanding they begin to solve pictorial doubling /halving problems.**

**As children gain in confidence they begin to respond rapidly to questions such as**

- **double 4**
- **half of 6**
- **two fives**
- **I roll double 3. What is my score?**
- **How many toes are there on two feet?**

- How many socks in 2 pairs.

These questions become progressively harder as the children gain in understanding.

## Year 2

Introduce counting in

Counting in 2's relate to odd and even

Counting in 10's flash ten fingers as they say the words

Counting in 5's flash five fingers as they say the words

Children begin to find  $\frac{1}{4}$ 's and  $\frac{1}{3}$ 's of amounts they do this practically using blocks and cubes etc.

**Introduce language of multiples and factors, times, groups of ,sets of etc**

Children begin to record calculations using x sign.

Children use counting fingers to show the multiples to go with the factors e.g. hold up 0 fingers for  $0 \times 3$ , 1 finger for  $1 \times 3$  etc

Children need to be show the multiples of a specified times table and to be able to identify the associated factors to go with.  
E.g. 30 6 and 5 if they are learning the  $5 \times$  table.


Children investigate times tables using array patterns

**Children in year 2** learn to count forwards and backwards in steps of 2,3,4,5 and 10.

They use a number line or a 100 square to help them.

### Example

0---1---2---3---4---5---6---7---8---9---10 counting in 2's on a number line



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

**Counting in 5's on a 100 square**

The children begin to recognise patterns. The more familiar they become with these patterns the easier they find it to multiply and divide. It is essential that they learn to count forwards as well as backwards.

**Children in year 3** learn to count in 6,7's,8's and 9's, but must continue to practice the other tables.

**Children in year 4,5 and 6** practice and apply all times tables.

Year 3,4,5,6

Counting in all tables both up and down

**Build up new tables on a blank number line using doubling, halving and combining.**

Children use counting fingers to show the multiples to go with the factors e.g. hold up 0 fingers for 0x3, 1 finger for 1x3 etc

Array patterns used to support their learning of the new tables being introduced.

0X3= 0  
1x3= 000 3  
2X3= 000 6  
3X3= 000 9  
4X3= 000 12  
5X3= 000 15

Children need to be show the multiples of a specified times table and to be able to identify the associated factors to go with.  
E.g. 30 6 and 5 if they are learning the 5 x table



Extend counting knowledge by counting in 19's 29's etc

### Multiplication/Division as an Array pattern

Array patterns must be used to teach x and -:- facts. They must not be used to teach just multiplication. Children will be expected to be able to say that if there are 2 rows of 5 cubes the associated facts are 5x2 2x5, 10-:-2 and 10-:-5

The children begin to look at array patterns in Year 2, this knowledge is extended and embedded in year 3 and should then be applied readily in year 5 and 6.

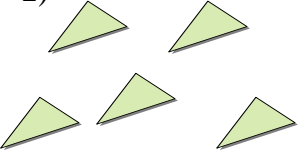
Example

 this array shows 2 rows of 5 squares or 10 divided between 2  
  $2 \times 5$  or  $10 \div 2$   $10 \div 5$   
 or 5 columns of 2  
 $5 \times 2$

Children are given tasks such as draw all the array patterns using 6 squares and write the multiplication and division sentence (the calculation) to accompany it.

### Example

1)  Can you share these cubes equally between two friends?

2)  Double the number of triangles drawn here.

### Multiplication as repeated addition

#### Multiplication calculations

In Reception and Year 1 the children begin to create repeating patterns using beads or pictures etc. This is the beginning of multiplication and division. They will be asked to solve problems in the role play area. For example the children may be 'playing' the Teddy Bears picnic with a teacher or classroom assistant. They could be told that there are 3 bears. Can they give each bear two sandwiches? Learning through play is crucial at this stage to set abstract concepts into realistic scenarios. This will provide the foundations for the later layers to be taught.

## Multiplication and division as repeated addition and subtraction using a number line

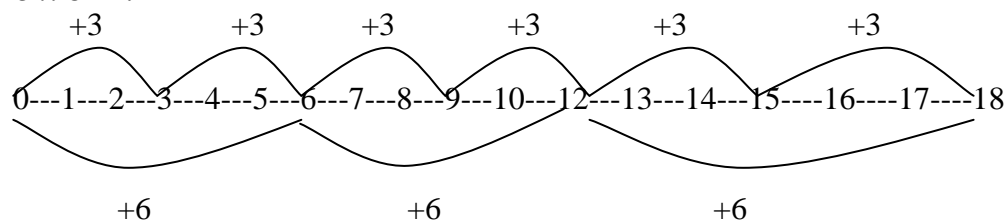
### Multiplication using a number line

Children in year 2 begin to multiply and divide using a number line with calculations based on the 2,3,4,5 and 10 times tables.

#### Example

Children start at 0 and count along in jumps as indicated in the number sentence. This method clearly shows that multiplication is repeated addition.

$$3 \times 6 = 18$$



$$6 \times 3 = 18$$

Using number lines like this allows the children to see the relationship between  $3 \times 6$  and  $6 \times 3$ .

### Pencil and paper procedures for multiplication

In year 3 children are introduced to the grid method of multiplication.

By the end of year 2 children must be secure in dividing and multiplying by 10 to enable them to do the grid method.

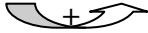
#### The grid method

Example 1) Multiplying one-digit numbers

$$\begin{array}{r} 4 \\ 3 \end{array} \boxed{12} \quad 3 \times 4 \text{ can be solved using a very simple grid}$$


**Example 2) Multiplying one-digit by two-digit numbers**

Before they can undertake one-digit by two-digit multiplication children must be secure in multiplying numbers by 10, 20, 30 etc

<b>X</b>	10	3	
3	<b>30</b>	<b>9</b>	$13 \times 3 = 30 + 9$ Product 39
			
	add together		

**Example 3) Multiplying two two-digit numbers.**

Ensure that the biggest box surrounds the tens numbers which are being multiplied. This provides the children with a visual clue that the product in that box needs to be of the greatest value.

<b>X</b>	<b>30</b>	<b>5</b>	
<b>10</b>	<b>300</b>	<b>50</b>	$35 \times 14 = 300 + 120 + 50 + 20$ Product 490
<b>4</b>			
			

Add together all the products



**Example 4) Multiplying a three-digit number by a two-digit number**

<b>X</b>	100	40	2
30	<b>3000</b>	<b>1200</b>	<b>60</b>
5	<b>500</b>	<b>200</b>	<b>10</b>

$142 \times 35 = 3000 + 1200 + 500 + 200 + 60 + 10$

**Product 4970**

**Partitioning**

Introduced in year 5 to the more able as an alternative method.

The grid method allows the children to see all of the stages of multiplication required.

This method of multiplication leads on from the grid method. The children need to be secure with the grid method before they attempt to record their calculations using this format.

	H T U
	2 3
	x 7
	-----
20 x 7	1 4 0
3 x 7	2 1
	-----
	1 6 1

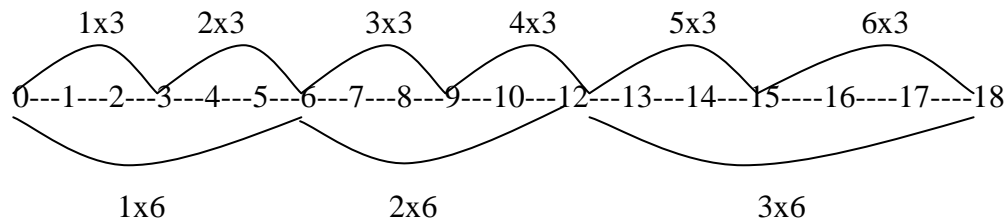
This method can be built up to multiply three and four-digit numbers. Initially it allows the children to use the grid method alongside while they begin to record using this method.

## Division using a number line

### Example

**Starting in year 3** division can be worked out using the same technique as multiplication, except this time the children start with the highest number, this tells them how many items in total which need to be shared out, they then count in equal jumps from 0 as indicated by the calculation. It is important that the children count up from 0 because when a remainder is involved the numbers would be in unfamiliar steps if they counted back.

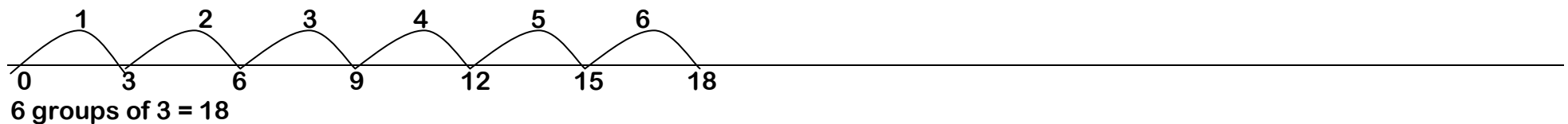
$$18 \div 3 = 6$$



$$18 \div 6 = 3$$

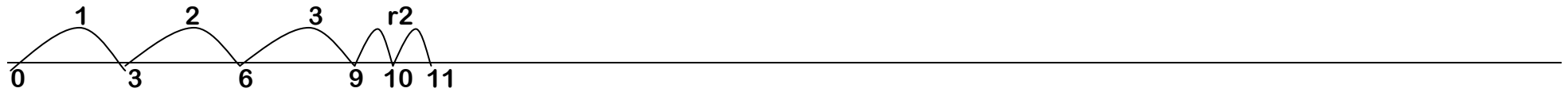
The above number line clearly shows the children the relationship between  $18 \div 3$  and  $18 \div 6$  and their multiplication tables.

The above diagram is how the teacher would model the process; children would use a more informal approach and tend to record their working out on a blank number line e.g.



## Division with a remainder

$$11 \div 3$$



## Grouping and sharing

**In reception** children learn about grouping and sharing through role play using toys etc here are 3 teddies we need to dress them how many socks do we need. Or we have 6 socks how many teddies will get socks?

**In year 1** the children learn the difference between grouping and sharing and understand that to share you go round one for you one for me etc whereas grouping is arranging a set of objects into equal groups of a given number.

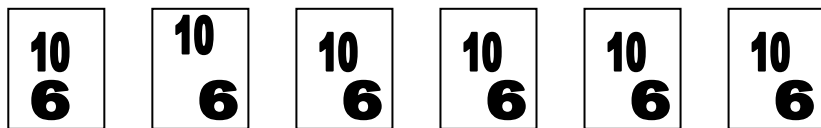
It is essential that grouping is taught **in years 2,3,4 and 5** and that children understand that this is different to sharing. In year 5 and 6 children need to be able to group to solve long division problems.

## Example

$$96 \div 6 =$$

This can initially be done through grouping.

The children decide it is too greater an amount to share out one for you one for you etc so they are encouraged to share out groups of 10 to each set.



Children then calculate that they have used 60 of the 96 items to be shared out and are therefore left with 36 items. These can then be grouped into 6's and shared out equally.

The children can then see that each set now contains 16 items and that the full 96 have been equally divided between 6.

This method is then used to help the children record their division calculations using the more formal written method listed below. This method needs only be taught to the more able yr5's. The rest of the class must be totally secure with grouping method.

Dealing with remainders.

The above method can be taught using remainders in exactly the same way.

The formal written method

Children are encouraged to approximate an answer. For example,  $96 \div 6$  is approximately  $100 \div 5 = 20$  or  $5 \times 20 = 100$

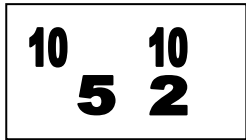
$$\begin{array}{r}
 \text{T U} \\
 16 \\
 \hline
 6 \overline{) 96} \\
 - 60 \quad 10 \times 6 \\
 \hline
 36 \quad (\text{left over}) \\
 - 36 \quad 6 \times 6 \\
 \hline
 0
 \end{array}$$

Answer: 16

Children need to make sure that their calculations are set out in columns, with the digits in the correct H T or U position and this must be taught alongside the grouping method until the children are secure with the written layout.

In year 6 children are expected to divide 3 digit by 2 digit numbers with remainders expressed as a fraction. Children can only do this if they are totally secure with the grouping method.

When dealing with large divisions children need to realise that they don't need to draw all 36 sets but that drawing just one of the 36 sets can aid as a prompt to their working out.



Just draw one of the 36 boxes

$$10 \times 36 = 360$$

$$10 \times 36 = 360 \quad \text{total} = 974$$

$$5 \times 36 = 180$$

$$2 \times 36 = 72$$

$$973 \div 36 = 27 \text{ r}1 \text{ or } 27 \frac{1}{36}$$

973  $\div$  36 is approximately 1000  $\div$  40 = 25

$$\begin{array}{r}
 \underline{27} \\
 36 \overline{) 973} \\
 \underline{- 360} \quad 10 \times 36 \\
 613 \\
 \underline{- 360} \quad 10 \times 36 \\
 253 \\
 \underline{- 180} \quad 5 \times 36 \\
 73 \\
 \underline{- 72} \quad 2 \times 36 \\
 1
 \end{array}$$

Answer : 27 and 1/36 or 27 r 1

For children to be able to do these long divisions they need to know that 10x an amount is double 5x an amount. This needs to be secure by the end of KS1.